Constraining Electron Parallel Energy in Electrostatic Fields through the Anomalous Doppler Effect Induced by External Electromagnetic Waves

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Abstract

The interaction between free electrons and electromagnetic waves (EMW) under the influence of magnetic and electrostatic fields is investigated using a Volume-Preserving algorithm. When the electric field of the EMW, containing a left-hand polarization component, exceeds a critical threshold, it facilitates continuous transfer of parallel electron energy into rotational energy through the Anomalous Doppler Effect (ADE). This process transforms the electric field's work along the magnetic field into perpendicular kinetic energy, leading to saturation of the electron’s parallel kinetic energy and continuous growth of its perpendicular kinetic energy. A theoretical model based on energy, momentum, and angular momentum conservation elucidates the role of left-hand polarization in the Anomalous Doppler Effect and provides a generalized framework for interpreting electron-wave interactions. This study proposes a novel approach for mitigating runaway electrons in magnetically confined plasmas, suggesting the use of extraordinary waves launched from the high-field side with an energy flux of hundred watts per square meter to saturate parallel energy in Tokamaks.

Keywords: runaway electrons, anomalous doppler effect, extraordinary wave, left-hand polarized wave

I. Introduction

In the beginning of burning plasma device discharge (current ramp up phase), the magnetohydrodynamic (MHD) instabilities and disruption can generate quasi-static toroidal electric fields that accelerate electrons to energies reaching several tens of MeV. This acceleration occurs when the force exerted by the quasi-static electric field surpasses the opposing forces from radiation and collisional drag. These high-energy electrons, known as runaway electrons, can inflict severe damage on the tokamak’s interior walls, thereby shortening the device’s operational lifespan. An intriguing possibility is to convert the energy gained by electrons from quasi-static electric fields into rotational energy within the magnetic field. This approach not only suppresses the energy of runaway electrons, reducing their harmful impact on the device, but also improves discharge performance by minimizing the consumption of ohmic field energy.

The transport of parallel energy from electrons into rotational energy primarily occurs through three different mechanisms, including (1) the electron avalanche process [[1](#_ENREF_1)], (2) collision less pitch-angle scattering[[2](#_ENREF_2)], and (3) the Anomalous Doppler Effect [[3](#_ENREF_3)]. Current strategies to suppress runaway electrons, such as gas injection [[4](#_ENREF_4)] and the enhancement of magnetic perturbations [[5](#_ENREF_5)], often have unintended side effects and disrupt the discharge environment. In contrast, the Anomalous Doppler Effect provides a cleaner mechanism, making it a particularly attractive avenue for further investigation.

When electrons move in static magnetic fields and interact with external electromagnetic waves (EMW) of frequency ω and wave vector k⃗, they undergo a scattering phenomenon under the resonant condition ω-k ⃗⋅v ⃗=mω\_ce, where m<0 and ω\_ce>0.This scattering results in the transfer of momentum from parallel motion to rotational motion, a phenomenon known as the Anomalous Doppler Effect. The Anomalous Doppler Effect was first thoroughly described in the seminal works of Ginzburg and Frank [[3](#_ENREF_3), [6](#_ENREF_6), [7](#_ENREF_7)].

Recently, the Anomalous Doppler Effect has garnered increasing attention in fields such as space radiation [[8](#_ENREF_8)] , runaway electron instabilities [[9](#_ENREF_9)], and materials science [[10](#_ENREF_10)]. It is believed that Anomalous Doppler Effect can explain phenomena like whistler turbulence in solar flare loops [[8](#_ENREF_8)], the step-like structure in Electron Cyclotron Emission (ECE) observed in tokamaks [[11-13](#_ENREF_11)], and the microwave bursts during Edge Localized Modes (ELMs) [[14](#_ENREF_14)]. Furthermore, Anomalous Doppler Effect has shown potential for suppressing runaway electron energy in tokamak discharges.

This potential was demonstrated by F. Santini [[15](#_ENREF_15)], who found that high-energy runaway electrons could be significantly reduced through Anomalous Doppler Effect during lower hybrid wave heating in the Frascati Tokamak . However, it is important to note that the high power of lower hybrid waves also increases the population of nonthermal electrons through Landau resonance, leading to a subsequent rise in runaway electrons after the lower hybrid waves are turned off. This side effect poses a challenge to the use of lower hybrid waves for suppressing runaway electrons.

Additionally, the experiment on electron beam energy transformation conducted by E.G. Shustin [[16](#_ENREF_16)] demonstrated that the transverse energy of electrons increases significantly through the Anomalous Doppler Effect when the beam excites waves within the frequency range between the electron cyclotron frequency and the upper hybrid frequency. This intrinsic plasma wave is generated via wave-particle interactions, resulting in the scattering of the electron beam's parallel velocity into the perpendicular direction. Furthermore, C. Liu [[9](#_ENREF_9)] investigated runaway kinetic instability using the kinetic equation and observed that when whistler waves are excited, they cause the scattering of runaway electrons via the Anomalous Doppler Effect. Similar findings include the runaway scattering effect observed on HT-7 [[17](#_ENREF_17)] and FTU [[18](#_ENREF_18)], as well as energetic electron scattering in solar flare loops [[8](#_ENREF_8)], among others. These waves can not only be generated through wave-particle interactions but also through external injection. Consequently, suppressing the parallel energy of runaway electrons through the Anomalous Doppler Effect by injecting specific electromagnetic waves appears to be a natural approach. While a previous study has proposed using whistler waves to suppress runaway electrons based on simulations with the quasilinear kinetic equation [[19](#_ENREF_19)], further exploration and investigation remain crucial to fully comprehend and optimize this approach for practical applications.

Understanding the Anomalous Doppler Effect in the presence of electrostatic fields is essential for comprehending the physics of pitch-angle scattering of runaway electrons by electromagnetic waves in Tokamak discharges. Additionally, the basic physics of Anomalous Doppler Effect remains intricacy to understand. It is still lacking a clear physical understanding of why parallel kinetic energy can convert into transverse internal energy during Anomalous Doppler Effect resonance, and what kind of electromagnetic waves can trigger velocity scattering. Despite the Anomalous Doppler Effect has been explored in either based on the test particle [[20](#_ENREF_20)] or quasilinear kinetic equation [[21-23](#_ENREF_21)], this question remains need to answer. In this paper, test electrons are used to investigate the Anomalous Doppler Effect in an effort to address the question outlined above. A theoretical model based on energy, momentum, and angular momentum conservation is proposed to explain the role of left-hand polarization in the Anomalous Doppler Effect. This model is provided in the appendix for reference to ensure that the main discussion in the paper remains cohesive and uninterrupted.

This paper presents a direct simulation of full orbit electron motion in uniform magnetic fields, along with accelerating electrostatic and electromagnetic fields, using the Volume-Preserving Algorithm [[24](#_ENREF_24)]. Compared to conventional algorithms like Boris [[25](#_ENREF_25)], the Volume-Preserving Algorithm ensures long-term accuracy and conservativeness through a systematic splitting method, making it an ideal approach for nonlinear electron dynamic simulations. To directly observe the Anomalous Doppler Effect, an electron is placed in a uniform magnetic field and an electrostatic field, which is oriented opposite to background magnetic field. This setup allows the electron to be accelerated parallel to background magnetic field. During the simulation, a slow electromagnetic wave with a phase velocity smaller than that of light in vacuum is introduced as an induced wave. This wave enables us to observe the effects when the electron’s velocity reaches the resonant condition for the Anomalous Doppler Effect. We explore resonance with three types of polarization waves: linear polarization, left-hand circular polarization, and right-hand circular polarization. The results show that only the wave with left-hand circular polarization induces the Anomalous Doppler Effect for runaway electrons. The simulation also reveals the critical energy of waves at which the electron's parallel velocity is constrained and consistently transfers parallel energy from the electrostatic field to transverse rotational energy. Furthermore, the self-consistency between quantum theory and direct simulation of the Anomalous Doppler Effect is examined. The analysis of dispersion, polarization, and resonant moments, we determine that the extraordinary wave is most suitable for triggering the Anomalous Doppler Effect in plasma. Based on these findings, we propose an effective method for controlling runaway electrons.

The numerical simulation framework and results are presented in Section II. The trapping threshold is examined in Section III. Section IV explores the dynamics of electromagnetic waves driving the Anomalous Doppler Effect in magnetized plasma. The runaway electron suppression method using extraordinary wave injection is introduced in Section V. Finally, the summary is provided in Section VI.

II. Numerical Simulation Framework and Result Discussion

The uniform magnetic field is set on the z-direction. The electron is accelerated by the electrostatic field , which on the opposite direction to as shown in Fig. 4. For the dynamics analysis of electrons during interactions with an electromagnetic field, a plane electromagnetic wave is established, which characterized by frequency ω and wavevector .

The electron orbit and motion p in this scenario is presented as Eq. 16. The E ⃗ and B ⃗ are the total field contains static field and electromagnetic field.

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Figure 1. The uniform background magnetic is set on z direction (orange). The electrostatic field is marked with green. The electromagnetic field propagates along z direction, with the linear polarization along x direction. The electron orbit has been plotted in black.

The discrete structure of eq. is rewritten as eq. by employing the Volume-Preserving Algorithm [[24](#_ENREF_24), [26](#_ENREF_26), [27](#_ENREF_27)]. The operator Cay(A) denotes the Cayley transform of matrix A [[24](#_ENREF_24)].

The dimensionless magnetic matrix \* is presented as Eq. 18.

The dimensionless parameters are momentum , magnetic field , total electric field , time step , and position respectively, where the is the electron cyclotron period and .

As a preliminary validation calculation, the parameters are set as follows. The background magnetic field . The wave angle frequency is , where . The wavevector . The amplitude of the electric field of the electromagnetic wave is , and the electrostatic field is . All these parameters are only set for the purpose of rapid simulation. The real tokamak scale calculation will be discussed in the following section. The time step is always chosen to satisfy 50()) to ensure the accuracy of the simulation. The electron begins at rest and gradually gains speed. The resonant frequency increases according to Eq. 13, 14, and 15.

A diagram of a coil with a wire

Description automatically generated with medium confidence

Figure 2. (left) Orbit trajectory of electron motion. (right) Momentum phase space of electron motion.

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Figure 3. Kinetic evolution of electrons in a magnetic field with electromagnetic wave during acceleration. (a) Wave frequencies of anomalous Doppler frequency, normal Doppler frequency, and source wave frequency. (b) The parallel velocity in the case with and without the electromagnetic wave. (c) The change of parallel velocity caused by the electromagnetic wave. (d) The cyclotron velocity v\_⊥. (e) The change of circular velocity during interaction with linear, right-hand circular, and left-hand circular polarization. (f) The change of parallel velocity during interaction with linear, right-hand circular, and left-hand circular polarization.

Figure 2 illustrates the evolution of the electron’s orbit and velocity phase during acceleration. The details of the electron’s motion are shown in figure 3. As the electron accelerates in the electrostatic field (figure 3(b)), the resonant frequencies increase concurrently (figure 3(a)). At around 23 , when the Normal Doppler Frequency matches that of the induced wave, the perpendicular velocity (or rotational velocity) increases rapidly (figure 3(d)). The parallel velocity induced by the electromagnetic wave also increases, as shown in figure 3(c). This change can be calculated as , where is the parallel velocity due to both the electromagnetic wave and the electrostatic field, and is the parallel velocity resulting only from the electrostatic field.

This phenomenon corresponds to the Normal Doppler Effect, where the resonant velocity is "subluminal." The absorption of induced waves by the cyclotron electron results in an increase in both parallel and perpendicular velocities, which can be considered a reverse process to the photon emission described in the appendix. The Normal Doppler Effect process is widely used for current drive [[28](#_ENREF_28)] and plasma heating [[29](#_ENREF_29)] in tokamaks. However, it is generally believed that current drive via electromagnetic waves follows the Fisch mechanism [[30](#_ENREF_30)], due to the limited toroidal momentum injected by the waves.

1.1 Subsection heading

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